Int. J. Heat Mass Transfer. Vol. 36, No. 16, pp. 3931–3935, 1993
Printed in Great Britain

0017-9310/93 $6.00 + 0.00

High frequency asymptotics of acoustical nucleate pool boiling noise spectrum in compressible viscous liquids

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(*Received* 3 *April* 1992 *and in final form* 29 *April* 1993)

Abstract—An expression for the high frequency asymptotics of the acoustic noise spectrum in nucleate pool boiling is derived taking into account the compressibility and viscosity effects. The expression is obtained in quasi-acoustical approximation. The theoretical results qualitatively agree with previous measurements of boiling noise.

INTRODUCTION

THE ACOUSTICAL emission from boiling bubbles has been studied experimentally and theoretically. These studies investigated the nucleate pool boiling which occurs when a heater is submerged in a tank of initially stagnant liquid. The spectra of acoustical noise in pool boiling were obtained [1–3], and it was established that the spectral density of boiling noise spectra varies as $S \sim f^4$ for low frequencies and $S \sim f^{-2}$ for high frequencies [4]. Relatively few detailed studies have been made for a theoretical definition of spectrum form. The works of Fitzpatric [5], Benjamin [6], Gilmore [7] may be mentioned, in which they considered isothermal cases connected with cavitation problems, and Likhterov and Elperin [8] where the process of sound generation was treated for cases of bubble growth and implosion. It has been shown that the high frequency part of the noise spectrum varies as $S \sim f^{-2/5}$, but such slow decrease of the spectral density stems from assumption of liquid incompressibility and neglecting dissipative effects. The present work was undertaken to consider these factors and to extend the information available on the sound emission in the nucleate pool boiling.

THEORETICAL ASPECTS

The momentum equation for compressible and viscous liquid is

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\,\mathrm{grad})\mathbf{V} = -\frac{1}{\rho}\mathrm{grad}\,p + \frac{\nu}{3}\mathrm{grad}\,\mathrm{div}\,\mathbf{V} + \nu\Delta\mathbf{V} \tag{1}$$

and continuity equation is

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{div}\,\mathbf{V}, \tag{2}$$

where $\mathbf{V}$ is velocity vector.

A problem under consideration has a spherical symmetry and consequently, velocities in all liquids have a potential $(r\phi)$, therefore

$$\mathrm{curl}\,\mathbf{V} = 0. \tag{3}$$

The last term in equation (1) can be represented as

$$\nu\Delta\mathbf{V} = \nu\,\mathrm{grad}\,\mathrm{div}\,\mathbf{V} - \nu\,\mathrm{curl}\,\mathrm{curl}\,\mathbf{V}$$

and because of equation (3), it is apparent that

$$\nu\Delta\mathbf{V} = \nu\,\mathrm{grad}\,\mathrm{div}\,\mathbf{V}, \tag{4}$$

and Navier–Stokes equations get simplified:

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\,\mathrm{grad})\mathbf{V} = -\frac{1}{\rho}\mathrm{grad}\,p + \frac{4}{3}\nu\,\mathrm{grad}\,\mathrm{div}\,\mathbf{V}. \tag{5}$$

The last term in equation (5) is a product of the viscosity kinematic coefficient and the value

$$\mathrm{grad}\left(\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}\right).$$

It equals zero in the case of inviscid or incompressible liquid. In the problem under consideration it may be supposed that viscosity is small and that compressibility is moderate, therefore this term can be neglected. The influence of the viscosity may be taken into consideration through boundary conditions. In spherical coordinates equation (5) will be

$$\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}. \tag{6}$$

Integrating (6) allows us to obtain

$$\frac{\partial \phi}{\partial t} + \frac{v_r^2}{2} = -\int_{p_\infty}^{p}\frac{\mathrm{d}p}{\rho} = -h(p). \tag{7}$$

It is assumed that $v_r = 0$ and $p = p_\infty$ at $r = \infty$.

Besides the equation (7) use is made of the Kirkwood and Bethe hypothesis [9] that

NOMENCLATURE

C	sound velocity on the bubble boundary
c_∞	sound velocity in undisturbed liquid
E	acoustical energy
f	frequency
h	enthalpy
l	specific enthalpy of vaporization
p	liquid pressure
p_∞	liquid pressure at great distance from bubble or ambient pressure
p_v	saturated vapor pressure
p_s	adiabatic acoustic pressure
R	bubble radius
$\dot{R}$	radial velocity of bubble boundary
$\ddot{R}$	radial acceleration at bubble boundary
R_0	equilibrium radius of bubble in liquid at uniform superheating
r	distance from bubble center
S	spectral density of the acoustical energy
s	entropy
T	initially uniform temperature of superheated liquid
T_R	instantaneous temperature at bubble boundary
T_s	saturated temperature of vapor
t	time
$\mathbf{V}$	liquid velocity vector
v_r	radial velocity.

Greek symbols

μ	dynamic viscosity coefficient
ν	kinematic viscosity coefficient
ρ	liquid density
ρ_∞	liquid density at great distance from bubble
ρ_v	saturated vapor density
σ	surface tension coefficient
ϕ	velocity potential.

$$\left[\frac{\partial}{\partial t}+(c+v_r)\frac{\partial}{\partial r}\right]\left[r\left(h+\frac{v_r^2}{2}\right)\right]=0, \tag{8}$$

where

$$\mathrm{d}h = T\mathrm{d}s+\frac{1}{\rho}\mathrm{d}p, \tag{9}$$

T is liquid temperature, s is entropy. For isentropic processes $\mathrm{d}s = 0$, as the thermal dissipation is very small even in the shock wave front.

The solution of (7)–(9) has been completed by Gilmore [7], with the assumption that in a viscous liquid one may write

$$p = p_v - \frac{2\sigma}{R}+2\mu\left(\frac{\partial v_r}{\partial r}\right)_{r=R}, \tag{10}$$

where p_v is the gas pressure inside the bubble.

The Tait equation connecting the pressure and density of the liquid for adiabatic (isentropic) process may be used:

$$\frac{p+B}{p_\infty+B}=\left(\frac{\rho}{\rho_\infty}\right)^n,$$

where $B = 3000$ atm and $n = 7$.

Sound velocity square is

$$c^2=\frac{\mathrm{d}p}{\mathrm{d}\rho}=\frac{n}{\rho}(p+B)=\frac{n}{\rho_\infty}(p_\infty+B)\left(\frac{p+B}{p_\infty+B}\right)^{1-1/n}$$

and at $p = p_\infty$

$$c_\infty^2=\frac{n}{\rho_\infty}(p_\infty+B).$$

The local sound velocity is then

$$c=c_\infty\left(\frac{p+B}{p_\infty+B}\right)^{1-1/n}.$$

The Gilmore solution for the bubble boundary velocity $\dot{R}$ in the case of bubble implosion [7] has following form:

$$\left(\frac{R_0}{R}\right)^3=\left(1-\frac{\dot{R}}{3C}\right)^4\left[1+\frac{3\rho_\infty\dot{R}^2}{2(p_\infty-p_v)}\right], \tag{11}$$

where

$$C=c_\infty\left(\frac{p+B}{p_\infty+B}\right)^{(n-2)/2n}$$

is sound velocity on the bubble boundary.

The vapor pressure in the bubble can be related to the vapor temperature by linearized Clapeyron equation [10]

$$p_\infty-p_v=\frac{\rho_v l}{T_s(p_\infty)}[T_R-T_s(p_\infty)]. \tag{12}$$

In the equilibrium conditions ($\ddot{R} = \dot{R} = 0$) the radius of the vapor bubble is given as

$$R_0=\frac{2\sigma T_s}{\rho_v l(T_R-T_s)}. \tag{13}$$

Equation (11) reduces to

$$\left(\frac{R_0}{R}\right)^3\approx\frac{3\rho_\infty\dot{R}^2}{2(p_\infty-p_v)}, \tag{14}$$

if $\dot{R} \ll C$ and a considered case corresponds to inviscid and incompressible liquid. Integrating (14) results in

$$R^{5/2}=\frac{5}{2}\sqrt{\left(\frac{R_0^3}{a}\right)}t,$$

where

$$a=\frac{3\rho_\infty}{2(p_\infty-p_v)}.$$

If the velocity of the bubble boundary is sufficiently high, equation (11) can be reduced to

$$\left(\frac{R_0}{R}\right)^3\approx\left(\frac{\dot R}{3C}\right)^4\frac{3\rho_\infty}{2(p_\infty-p_v)}\dot R^2=\frac{\rho_\infty\dot R^6}{54(p_\infty-p_v)C^4},\tag{15}$$

and then on the assumption $C\approx c_\infty$

$$\frac{R_0}{R}=\dot R^2\sqrt[3]{\left(\frac{\rho_\infty}{54(p_\infty-p_v)c_\infty^4}\right)}$$

or

$$\dot R=\sqrt{\left(\frac{R_0}{a}\right)}R^{-1/2},\tag{16}$$

where

$$a=\sqrt[3]{\left(\frac{\rho_\infty}{54(p_\infty-p_v)c_\infty^4}\right)},\tag{17}$$

equation (16) may be integrated easily:

$$\dot R=\sqrt{\left(\frac{R_0}{a}\right)}R^{-1/2}\quad\text{or}\quad\frac{\mathrm{d}R}{\mathrm{d}t}=\sqrt{\left(\frac{R_0}{a}\right)}R^{-1/2}$$

and

$$R=\left(\frac{3}{2}\right)^{2/3}\left(\frac{R_0}{a}\right)^{1/3}t^{2/3}=At^{2/3},\tag{18}$$

where

$$A=3\times32^{-1/9}R_0^{1/3}c_\infty^{4/9}(p_\infty-p_v)^{1/9}\rho_\infty^{-1/9}.\tag{19}$$

The quasi-acoustical approximation may be used for definition of velocities and pressures. According to the approximation made by Trilling [11], it is assumed that the velocity potential will satisfy the acoustical equation

$$\left[\frac{\partial}{\partial t}+c_\infty\frac{\partial}{\partial r}\right](r\phi)=0,\tag{20}$$

and the momentum equation (1) can be written as

$$\frac{\partial v_r}{\partial t}+v_r\left(\frac{\partial v_r}{\partial r}\right)+\frac{1}{\rho}\left(\frac{\partial p}{\partial r}\right)=0.\tag{21}$$

Taking into account that $v_r=(\partial\phi/\partial r)$ equation (21) may be integrated on variable r:

$$\frac{\partial\phi}{\partial t}+\frac{v_r^2}{2}+\int_{p_\infty}^{p}\frac{\mathrm{d}p}{\rho}=0.\tag{22}$$

Since

$$\left.\begin{aligned}v_r&=\frac{\partial\phi}{\partial r}\\ r\phi&=g\left(t-\frac{r-R}{c_\infty}\right)\end{aligned}\right\},$$

where g is an arbitrary function, and the local liquid velocity will be

$$v_r=-\frac{g\left(t-\dfrac{r-R}{c_\infty}\right)}{r^2}-\frac{g'\left(t-\dfrac{r-R}{c_\infty}\right)}{rc_\infty}.\tag{23}$$

If the density ρ will be changed by ρ_∞ in the first order approximation, then

$$h=\int_{p_\infty}^{p}\frac{\mathrm{d}p}{\rho_\infty}=\frac{p-p_\infty}{\rho_\infty}$$

and on the basis of equation (7), the pressure may be expressed as

$$p=p_\infty-\frac{\rho_\infty g'\left(t-\dfrac{r-R}{c_\infty}\right)}{r}-\frac{\rho_\infty}{2}v_r^2.\tag{24}$$

The solution (23) and (24) in terms of an unknown function g and its derivative g' results

$$\left.\begin{aligned}g\left(t-\frac{r-R}{c_\infty}\right)&=\frac{r^2}{c_\infty}\left(c_\infty v_r-\frac{v_r^2}{2}-\frac{p-p_v}{\rho_\infty}\right)\\ g'\left(t-\frac{r-R}{c_\infty}\right)&=-r\left(\frac{v_r^2}{2}-\frac{p-p_v}{\rho_\infty}\right)\end{aligned}\right\}.\tag{25}$$

On the boundary of the bubble, where $r=R$, $v_r=\dot R$ and $p=p_v-(4\mu\dot R/R)$ (neglecting surface tension), these functions will be

$$\left.\begin{aligned}g(t)&=-\frac{R^2}{c_\infty}\left(c_\infty\dot R-\frac{\dot R^2}{2}+\frac{p_\infty-p_v}{\rho_\infty}+\frac{4\mu\dot R}{\rho_\infty R}\right)\\ g'(t)&=-\frac{R\dot R^2}{2}+\frac{R(p_\infty-p_v)}{\rho_\infty}+\frac{4\mu\dot R}{\rho_\infty}\end{aligned}\right\}.\tag{26}$$

Now, substitution of the expressions (23) and (26) into (24) allows us to obtain

$$\begin{aligned}p-p_\infty=p_s&\approx-\frac{\rho_\infty}{r}g'-\frac{\rho_\infty}{2}\left[\frac{g}{r^2}+\frac{g'}{rc_\infty}\right]^2\\ &\approx-\frac{\rho_\infty}{r}g'-\frac{\rho_\infty g'^2}{2r^2c_\infty^2}.\end{aligned}\tag{27}$$

The last term in this expression can be omitted retaining only first order terms, since acoustical pressures are considered for great distances from a bubble. Then the adiabatic acoustical pressure is in the form

$$p_s\approx\frac{\rho_\infty}{r}\left[\frac{R\dot R^2}{2}-\frac{R(p_\infty-p_v)}{\rho_\infty}+\frac{4\mu\dot R}{\rho_\infty}\right].\tag{28}$$

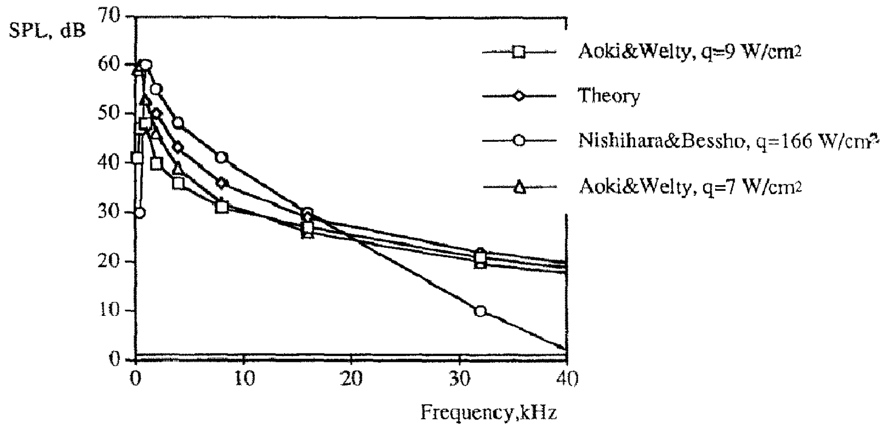

FIG. 1. Comparison of experimental and theoretical data.

Substitution of the expression (16) into (28) and exclusion of constant pressure components yield

$$p_s \approx -\frac{1}{r}\left[R(p_\infty - p_v) - 4\mu\sqrt{\left(\frac{R_0}{a}\right)}R^{-1/2}\right]. \tag{29}$$

The total acoustical energy emitted by the bubble is

$$E = \frac{4\pi r^2}{\rho_\infty c_\infty}\int_t^0 p_s^2\,\mathrm{d}t, \tag{30}$$

where (neglecting the second order infinitesimals)

$$p_s^2 \approx \frac{1}{r^2}\left[(p_\infty - p_v)^2 R^2 - 8\mu\sqrt{\left(\frac{R_0}{a}\right)}R^{1/2}\right]. \tag{31}$$

On the other hand, the emitted acoustical energy can be also defined as

$$E = \int_0^{\infty} S\,\mathrm{d}f, \tag{32}$$

where S is the spectral density of energy and f is the frequency.

Comparing the right hand sides of expressions (30) and (32) and taking into account that $f = 1/t$ allows us to obtain

$$\lim_{f\to\infty}\int_0^f S(f)\,\mathrm{d}f = -\frac{\rho_\infty c_\infty}{4\pi r^2}\left[(p_\infty - p_v)^2 A^2 f^{-7/3} - 8\mu\sqrt{\left(\frac{R_0}{a}\right)}A^{1/2}f^{-4/3}\right],$$

which after differentiating yields the expression for the spectral density

$$S(f) = \frac{\rho_\infty c_\infty}{4\pi r^2}[21\times 2^{-10/9}R_0^{2/3}c_\infty^{8/9} \times (p_\infty - p_v)^{20/9}\rho_\infty^{-2/9}f^{-10/3} + 2^{44/9} \times \mu R_0^{2/3}c_\infty^{8/9}(p_\infty - p_v)^{2/9}\rho_\infty^{-2/9}f^{-7/3}]$$

or

$$S(f) = \frac{\rho_\infty^{7/9}c_\infty^{17/19}R_0^{2/3}}{4\pi r^2}[9.72(p_\infty - p_v)^{20/9}f^{-10/3} + 29.5\mu(p_\infty - p_v)^{2/9}f^{-7/3}], \tag{33}$$

where the equilibrium size of a vapor bubble R_0 is given by expression (13) and saturated vapor pressure $p_v = p_v(T_0)$ can be taken from ref. [12].

CONCLUSION

The expression for the high-frequency part of the noise spectrum obtained in this work differs from the results obtained in ref. [8], where the spectral density of the energy for boiling noise decreased proportionally $f^{-2/5}$. It follows from equation (33) that taking into account the liquid compressibility and viscosity gives the stronger dependence for spectral density vs the frequency, i.e. $S(f) \sim f^{-7/3}$, as the second term of the expression (33) will prevail at big frequency values. It may be seen that the falldown of the spectral density is approximately of 10 $\log_{10}2^{-7/3} = 7$ **dB** octave^{-1} in the region of high frequencies is in agreement with the results of Nishihara and Bessho [3]. The comparison of experimental and theoretical data is presented in Fig. 1. At the same time substituting the superheat $T - T_s$ into the expression for boiling spectra (33) yields that the boiling noise intensity varies as $(T - T_s)^{-2/3}$, i.e. decreases with an increase of the superheat in an agreement with experimental data of Aoki and Welty [2].

Acknowledgement—This study was supported in part by a grant 3531-1-91 from the Israeli Ministry of Science and Technology.

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